A Computer Program For Decision Rules

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In order to understand optimal decision-making about saving, credit use, investing, insurance, and other financial decisions, it is useful to understand decision rules, such as choosing the action with the highest expected value or utility. To understand why people might make decisions inconsistent with rational rules, it is useful to consider sub-optimal rules, such as the MaxiMin, MaxiMax, and MiniMax Regret rules (Maurice & Thomas, 1999, pp. 686-689). A decision can be expressed in terms of actions, states of the world, and wealth levels in each action-state combination, as well as probabilities of each state of the world. We have developed a computer program that calculates the best action based on the MaxiMax, Best of the Most Probable, MaxiMin, MiniMax Regret and Laplace decision rules. The program also calculates the best action for the highest expected utility and expected value decision rules for a range of levels of risk aversion (Hanna & Chen, 1997).

Literature Review

Consumers face many decisions involving risk (Jensen 1986), including health and safety decisions, choice of occupations, information search, and most financial management decisions. The most powerful normative model for decision making with uncertainty is the expected utility model (Schoemaker 1982). Some researchers claim even when consumers are exposed to all of the information necessary to maximize expected utility in relatively simple problems, many consumers act irrationally and in an inconsistent manner (Kunreuther, 1976; 1978; Machina, 1987; Tversky & Kahneman, 1987). Although the expected utility model, and even more general 'logic of choice' models may not "... provide an adequate foundation for a descriptive theory of decision making" (Tversky & Kahneman 1987, p. 68), the expected utility model does provide a plausible basis for a normative model for consumer education, government policy, and even computer-based expert systems in the future.

Much has been written on the topic of the expected utility model and risky decisions, but much of the literature is too abstract for direct use in consumer education and public policy discussions (Arrow 1971) or, in the case of the standard geometrical exposition (Green 1978, p. 228, Deaton & Muellbauer 1980, p. 397), too vague to provide specific guidance. The lessons which can be provided by expected utility theory do not seem to be used sufficiently by consumer educators.

Illustration of Risky Choice

To simplify explanations, all examples will be for a model with only two possible states of the world and two possible actions. One action will give the consumer certainty in terms of wealth, e.g., a completely safe investment, such as a Certificate of Deposit (CD) while the other possible action could have two possible states of the world. The simplified situation is shown in the decision matrix below:

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Table 1
Wealth of an Individual, Depending on Action (invest in stocks or CD) and State of the World (stock market goes up or down)

	States of the World	
Actions	S1: market up	S2: market down
A1: 100% stocks	800,000	300,000
A2: 100% CD	400,000	400,000

If the individual invests everything in stocks and State of the World 1 occurs, wealth will be $800,000, but if State of the World 2 occurs, the consumer's wealth will be only $300,000. If the individual chooses Action 2, wealth will be $400,000 regardless of the State of the World. How should an individual decide which action is better? There are many possible non-rational decision rules, so-called because they do not take estimates of the probability of different states of the world into account, and one general rational rule, the Expected Utility rule.

Definitions

State of World: Future events or conditions that can influence on outcome or payoff but which cannot be controlled by the decision maker.
MaxiMax rule: Identify the best outcome for each possible decision and choosing the decision with the maximum payoff of all the best outcomes. This rule is also called the Optimist's rule.
MaxiMin rule: Identify the worst outcomes for each decision and choose the decision with the best worst payoff. This is also call the Pessimist's rule, because the decision-maker assumes the worst outcome of each action will happen.
MiniMax Regret rule: For each state of nature, when different actions are taken, there is a potential loss comparing to the payoff of the best action under that state of nature. A regret value is computed for each action under that state of nature. Choose the decision with the minimum of maximum regret values for each action. This rule is also called the rule for the regretful. (See Appendix for details.)
Laplace rule: Assuming that each state of nature is of equal probability, choose the action with the highest average payoff. This rule is also called ignorance rule.
Highest Expected Value rule: Choose the action with the highest expected value. The expected value is the sum of the value of each outcome times the probability of that outcome. This rule is same as the Highest Expected Utility rule, with a utility function with risk neutrality.
Utility Function: A mathematical function relating a person’s satisfaction to consumption (or wealth, as it represents all future potential consumption.) For analysis of decisions related to uncertainty, utility is considered to be a function of the level total consumption or wealth in each state of the world or time period.
Highest Expected Utility rule: The expected utility of each action is the product of the utility of each state of the world and the probability of each state of world. Choose the action with the highest expected utility.
Certainty Wealth Equivalent: The level of certain wealth that would produce the same utility as the expected wealth for uncertainty and a particular utility function.
Relative Risk Aversion: A measure of the relationship between utility and uncertainty. For a risk neutral person, relative risk aversion equals zero. For a risk seeking person, relative risk aversion is negative. For someone with a square root utility function, relative risk aversion = 0.5.

Table 2
Example of a Decision Matrix

	States of the World		Expected Value	Certainty Wealth Equivalent
Actions	S1: market up	S2: market down		
A1: 100% stocks	800,000	300,000	$700,000	$411,506
A2: 100% CD's	400,000	400,000	$400,000	$400,000
probability	0.8	0.2		rel. risk av. =6

In the example shown in Table 2, states of the world are defined in terms of whether the market is up or down. Only 2 states of the world are shown for simplicity, but any number can be defined. For instance, Hanna and Chen (1997) use over 50 states of the world. In this example (Table 2) the expected value of Action 1, a 100% stock portfolio, is $700,000 (0.8 x 800000 + 0.2 x 300000.) The expected value of Action 2, a portfolio composed of a perfectly safe investment such as Certificates of Deposits or Inflation Linked Bonds, is $400,000. A risk neutral person should choose Action 1.

Calculating the best action for each decision rule is somewhat easy with a two by two decision matrix, but may be difficult with many actions and many states of the world. The Payoff Program developed by Sherman Hanna can make it easier to find the best actions. For instance, for the example above, the Payoff Program produces the following output for the non-rational decision rules:

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  LAPLACE DECISION RULE
  IF ALL STATES OF THE WORLD ARE EQUALLY LIKELY, ACTION #  1
  WILL YIELD THE HIGHEST EXPECTED PAYOFF,     550,000.00

  MAXIMAX RULE: PAYOFF OF           800,000     BEST CHOICE IS ACTION    1

  MAXIMIN RULE: PAYOFF OF           400,000     BEST CHOICE IS ACTION    2

  REGRET VALUES
   STATE             1              2
  ACTION   1        0.0         100,000
       MAXIMUM REGRET=           100,000
  ACTION   2     400,000.0         0.0
       MAXIMUM REGRET=           400,000
  MINIMAX REGRET RULE: BEST CHOICE IS ACTION   1

 EXPECTED VALUES ARE:
  FOR ACTION   1:          700,000
  FOR ACTION   2:          400,000
  ACTION  1  HAS THE HIGHEST EXPECTED VALUE

  ACTION  1  IS BEST IF DECISION MAKER HAS RELATIVE RISK AVERSION OF   0.5
  RISK AVERSION IS VERY SLIGHT
  CERTAINTY WEALTH EQUIVALENT OF  BEST ACTION, #  1  =     680,767.3
  CERTAINTY WEALTH EQUIVALENT OF WORST ACTION, #  2  =     400,000.0

  ACTION  1  IS BEST IF DECISION MAKER HAS RELATIVE RISK AVERSION OF   0.9
  RISK AVERSION IS SLIGHT
  CERTAINTY WEALTH EQUIVALENT OF  BEST ACTION, #  1  =     662,480.8
  CERTAINTY WEALTH EQUIVALENT OF WORST ACTION, #  2  =     400,000.1

  ACTION  1  IS BEST IF DECISION MAKER HAS RELATIVE RISK AVERSION OF   1.0
  RISK AVERSION IS VERY MODERATE
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CERTAINTY WEALTH EQUIVALENT OF BEST ACTION, # 1 = 657,500.7
CERTAINTY WEALTH EQUIVALENT OF WORST ACTION, # 2 = 400,000.0

ACTION 1 IS BEST IF DECISION MAKER HAS RELATIVE RISK AVERSION OF 2.0
RISK AVERSION IS MODERATE
CERTAINTY WEALTH EQUIVALENT OF BEST ACTION, # 1 = 600,000.0
CERTAINTY WEALTH EQUIVALENT OF WORST ACTION, # 2 = 400,000.0

ACTION 1 IS BEST IF DECISION MAKER HAS RELATIVE RISK AVERSION OF 3.0
RISK AVERSION IS 'AVERAGE'
CERTAINTY WEALTH EQUIVALENT OF BEST ACTION, # 1 = 536,656.3
CERTAINTY WEALTH EQUIVALENT OF WORST ACTION, # 2 = 400,000.0

ACTION 1 IS BEST IF DECISION MAKER HAS RELATIVE RISK AVERSION OF 4.0
RISK AVERSION IS HIGH
CERTAINTY WEALTH EQUIVALENT OF BEST ACTION, # 1 = 481,286.9
CERTAINTY WEALTH EQUIVALENT OF WORST ACTION, # 2 = 400,000.0

ACTION 1 IS BEST IF DECISION MAKER HAS RELATIVE RISK AVERSION OF 5.0
RISK AVERSION IS VERY HIGH
CERTAINTY WEALTH EQUIVALENT OF BEST ACTION, # 1 = 440,147.4
CERTAINTY WEALTH EQUIVALENT OF WORST ACTION, # 2 = 400,000.0

ACTION 1 IS BEST IF DECISION MAKER HAS RELATIVE RISK AVERSION OF 6.0
RISK AVERSION IS EXTREMELY HIGH
CERTAINTY WEALTH EQUIVALENT OF BEST ACTION, # 1 = 411,506.1
CERTAINTY WEALTH EQUIVALENT OF WORST ACTION, # 2 = 400,000.0

ACTION 2 IS BEST IF DECISION MAKER HAS RELATIVE RISK AVERSION OF 10.0
RISK AVERSION IS INCREDIBLY HIGH
CERTAINTY WEALTH EQUIVALENT OF BEST ACTION, # 2 = 400,000.0
CERTAINTY WEALTH EQUIVALENT OF WORST ACTION, # 1 = 358,720.6

ACTION 2 IS BEST IF DECISION MAKER HAS RELATIVE RISK AVERSION OF 12.0
RISK AVERSION IS UNBELIEVABLY HIGH
CERTAINTY WEALTH EQUIVALENT OF BEST ACTION, # 2 = 400,000.0
CERTAINTY WEALTH EQUIVALENT OF WORST ACTION, # 1 = 347,264.8

A relative risk aversion level of 3.0 is comparable to the level of a person retiring at age 50 who is indifferent between taking a certain real aftertax income retirement of $50,000 per year for the rest of her life and a 50% chance of $100,000 per year and 50% chance of $37,976 per year for the rest of her life.

The Expected Utility Model

In general, expected utility EU is equal to the sum of the terms p_i times $U(W_i)$, where p_i is the probability of State of the World i, and W_i is the individual's wealth if that state of the world occurs. The sum of the probabilities must total one. For the purposes of this poster and in the novice mode of the Payoff Program, we assume that utility is a function of wealth, that utility always increases with wealth, but that the marginal utility of wealth decreases as wealth increases. The goal of the consumer is to maximize expected lifetime utility. Utility is assumed to be a function of wealth. The definition of wealth can be a sticky issue, but initially we can think of a retired person whose only source of income is perfectly liquid, perfectly safe financial assets.

<u>Relative Risk Aversion</u>

Utility functions can be characterized in terms of relative risk aversion, which is the same as the wealth elasticity of the marginal utility of wealth (Blume & Friend 1975). Relative risk aversion can be plausibly assumed to be constant for any person over usual ranges of wealth. Kimball (1988) has provided a nice intuitive illustration of the concept of relative risk aversion, appropriate for utility functions with constant relative risk aversion.

A modified version of Kimball's (1988) example is as follows: Assume that you are retiring at age 60 in a country with no social security system and no welfare, and that once you retire it will be impossible for you to obtain income from any other source. You have no assets of any kind. You may choose one of two retirement plans: A or B. Plan A pays you a tax-free real income of $50,000 per year forever, while Plan B involves a gamble. If you choose Plan B, the government in effect flips a coin, and you will have a fifty percent chance of having a real income of $100,000 tax-free forever, and a fifty percent chance of some lower income I. What is the lowest value of I for which you would prefer Plan B over Plan A? Table 3 shows how this level relates to relative risk aversion. Based on this thought experiment, most people will have a level of relative risk aversion in the range of 2 to 10.

Table 3

Modified Kimball Example of Relative Risk Aversion

Relative risk aversion	0	1	2	3	4	6	10	20
Lowest value of I	0	25,000	33,333	37,796	40,458	43,665	46,299	48,209

Economists have empirically estimated average values of relative risk aversion ranging from about one to over 10. In the context of the expected utility model, relative risk aversion relates to the extra utility of increased consumption if the gamble pays off compared to the lost utility because of decreased utility if you lose the gamble. For instance, if you have a relative risk aversion level of 4, you value the gain of utility from increasing your consumption from $50,000 to $100,000 the same as the loss of utility from decreasing your income from $50,000 to $40,548. The average utility gained per dollar if you choose Plan B and win is about 19 percent of the average utility lost per dollar if you lose. It seems plausible that most people in affluent countries have levels of relative risk aversion of at least one, but not much more than six. Friend and Blume (1975) suggest that their analysis of 1962 Survey of Consumer Finance data implies an average level of relative risk aversion in excess of two, assuming that investors were informed, rational, etc.

In calculating expected utility, the definition of wealth becomes especially important if the possibility of a substantial loss is considered. It is common to only enter the amount that might be put at risk in a utility function or decision matrix, but from the perspective of economic theory, that is clearly wrong, since utility should be viewed as being a function of all future consumption, including the part that might result from future earnings. A loss of $1,000 in the stock market is different for someone who has $1,000,000 in total wealth than for someone who only has 10,000 in total wealth. Furthermore, a $1,000 loss to a 25 year old with good earnings prospect should mean little compared to a $1,000 loss to an 80 year old. Therefore, human capital should always be added in to the wealth level in each State of the World/ Action combination. As Lee and Hanna (1995) reported, financial assets represent a small proportion of total household wealth for most U.S. households.

In terms of the example shown in Tables 1 and 2, even a risk averse person should choose Action 1, if relative risk aversion is less than 7 (assuming a constant relative risk aversion utility function.)
Composition

Maximin
regret

why expected utility rule is best, e.g., Maximin rule would lead to same result whether p of S2 = 20% or 0.00002%.

Appendix: Calculation of the MiniMax Regret Matrix

MiniMax Regret: Construct a regret matrix. Choose action which MINImizes the MAXimum regret.

Example: Payoff Matrix with 3 Actions and 3 States of the World

	State 1	State 2	State 3
Action 1	$W_{1,1}$	$W_{1,2}$	$W_{1,3}$
Action 2	$W_{2,1}$	$W_{2,2}$	$W_{2,3}$
Action 3	$W_{3,1}$	$W_{3,2}$	$W_{3,3}$

To construct a regret matrix, first work on each state of the world separately.

If $W_{m,1}$ is the maximum payoff for State of the World 1, then the regret for Action 1, State 1 is $W_{m,1}$ - $W_{1,1}$. The regret matrix is:

	State 1	State 2	State 3
Action 1	$W_{m,1} - W_{1,1}$	$W_{m,2} - W_{1,2}$	$W_{m,3} - W_{1,3}$
Action 2	$W_{m,1} - W_{2,1}$	$W_{m,2} - W_{2,2}$	$W_{m,3} - W_{2,3}$
Action 3	$W_{m,1} - W_{3,1}$	$W_{m,2} - W_{3,2}$	$W_{m,3} - W_{3,3}$

Find the maximum regret for each action, then choose the action with the lowest value of maximum regret.

References

Arrow, Kenneth J. (1971), *Essays in the theory of risk-bearing*, Chicago: Markham Publishing Co.

Bailey, M. J., Olson, M., & Wonnacott, P. (1980), The marginal utility of income does not increase: Borrowing, lending, and Friedman-Savage gambles, *American Economic Review*, 70 (No. 3; June): 372-379.

Deaton, A. & Muellbauer, J. (1980), *Economics and consumer behavior*, New York: Cambridge University Press.

Friedman, Milton and Leonard J. Savage (1948), The utility analysis of choice involving risk, *Journal of Political Economy*, 56 (No. 4: August): 279-306.

Friend, I. & Blume, M. E. (1975), The demand for risky assets, *American Economic Review*, 65 (Dec., no. 5):900-922.

Green, H. A. John (1978), *Consumer theory*, New York: Academic Press, Inc.

Hanna, S. & Chen, P. (1997). Subjective and objective risk tolerance: Implications for optimal portfolios. *Financial Counseling and Planning,* 8(2), 17-26.

Lee, H. & Hanna, S. (1995). Investment portfolios and human wealth, *Financial Counseling and Planning*, *6*, 147-152.

Machina, M. J. (1987), Choice under uncertainty: problems solved and unsolved, *Journal of Economics Perspectives*, 1 (No. 1; Summer): 121-155.

Maurice, S. C. & Thomas, C. R. (1995). *Managerial economics*. Chicago: Irwin.

Schoemaker, Paul J. H. (1982), The expected utility model: its variants, purposes, evidence and limitations, *Journal of Economic Literature*, 20 (No. 2; June): 529-63.

Tversky, Amos and Daniel Kahneman, Rational choice and the framing of decisions, in Robin M. Hogarth and Melvin W. Reder (eds.), Rational choice: the contrast between economics and psychology, Chicago: University of Chicago Press.